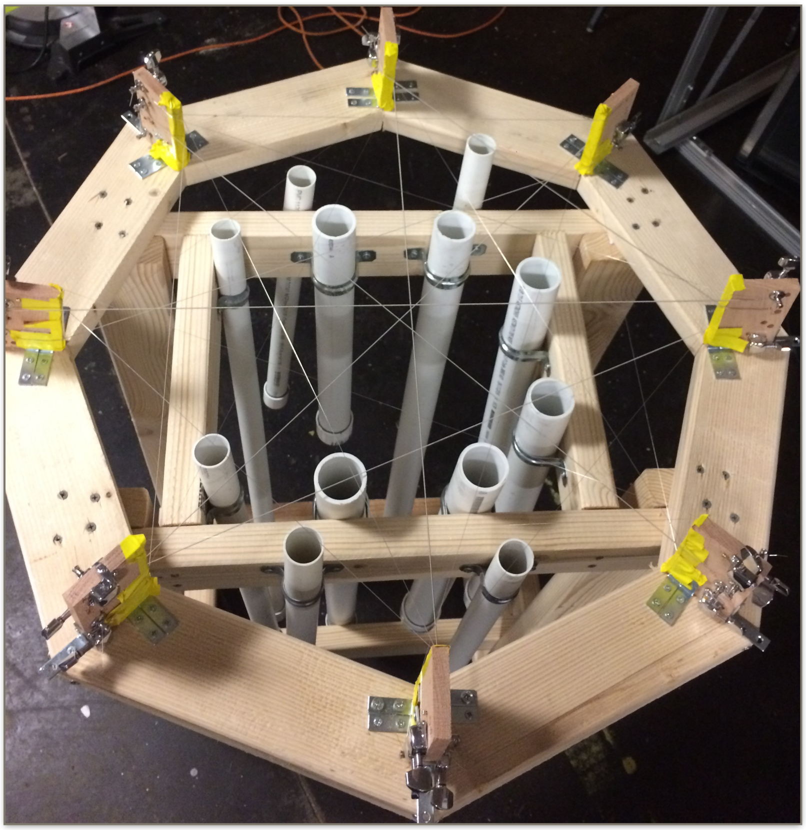
The Octagonal Harp

Music 8903 Design Project - Prof. Hsu



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Introduction

The octagonal harp is just as its name suggests, a harp that consists of the geometrical diagonals of an octagon. The original harp instrument dates back to the beginnings of civilization, and the octagonal harp brings a new innovation to the instrument with its overlapping strings. The octagonal features a balanced level of playing complexity and musical range.

Background

The design of the octagonal harp was inspired by the circular harp [1], which is a similar instrument that uses twelve points from which strings are drawn together. The circular harp has a resonance plate with a hole in it, similar to the resonating body of an acoustic guitar. As an interesting change, the octagonal harp has open-closed pipes below intersections of strings that resonate as the greatest common denominator frequency of the intersecting strings. This way, the instrument uses the acoustical power of the resonating strings and the resonating pipes for added volume and a different sound than vibrating strings alone. The player then stands over the harp, and plucks the strings they wish to play just as a harp player would play their instrument.

The octagonal harp incorporates 20 different diagonals of the octagon, all those diagonals that do not connect with an adjacent point of the octagon. This allows for a little more than one and a half octave range of the instrument. More strings would require more complexity of the instrument, which also increases the learning curve.

Design Overview

The design of the instrument incorporates some graph theory, geometry, and acoustic principals. The graph theory describes the number of levels of the strings on the instrument allowable without intersecting strings. Intersecting strings would cause the strings to prevent each other from vibrating. The geometry of the design allows for string frequencies that are fifths apart to intersect at different levels, an optimal place for a resonance pipe placement. The acoustic principals of the instrument dictate the string frequencies and the pipe resonance frequencies. The three design principals together create the instrument.

Graph Theory Principals

Given all intersections of the diagonals of an octagon, the result is depicted in **figure 1**.

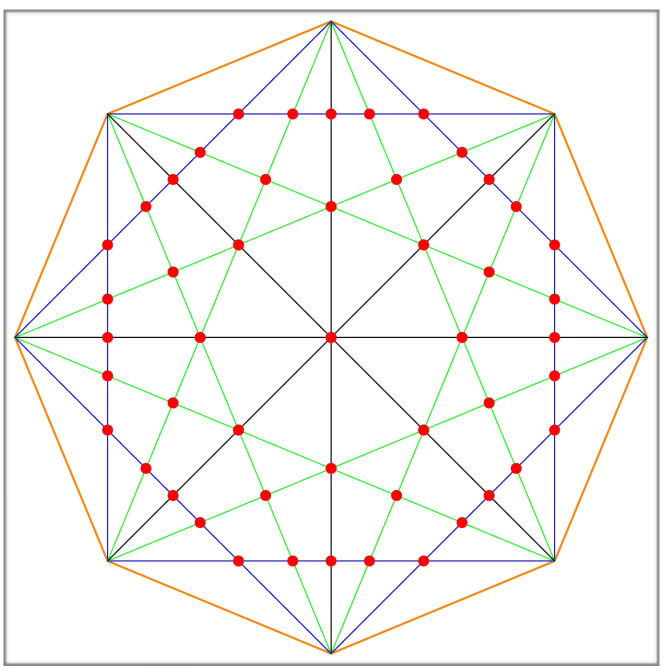

Fig 1. Diagonals and intersections of an octagon

There are 49 total intersections of the 20 diagonals. Limiting straight line connections from one note to another, and excluding connections that are made outside the shape of the octagon, it is possible to limit the amount of vertical levels to four, with five strings per level. This system is described in **figure 2**, with each different color referring to a different vertical level of the system.

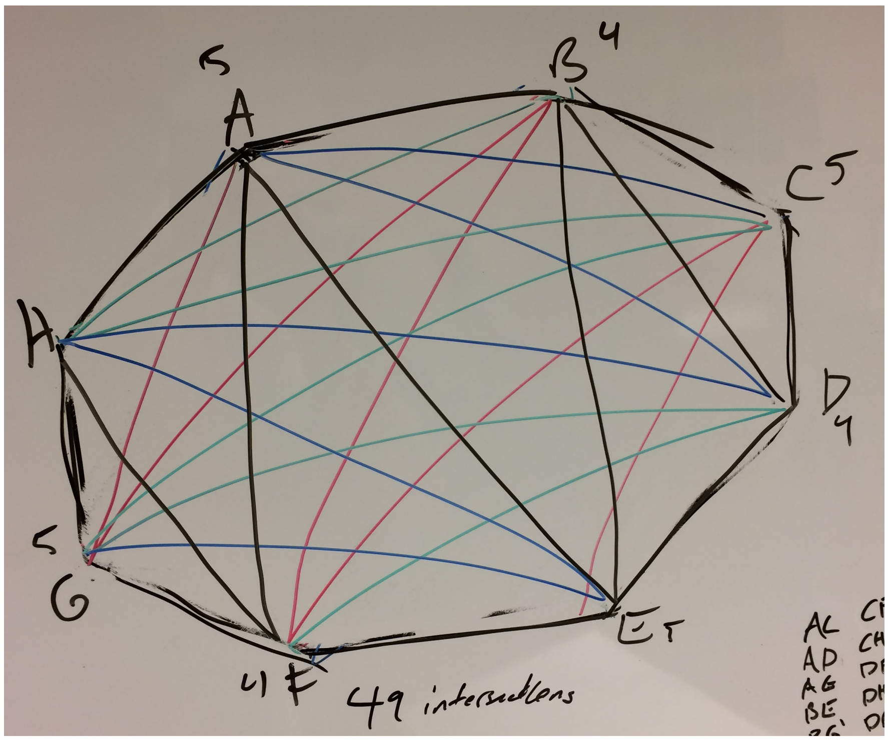

Fig 2. Intersections of an octagon efficiently sorted by level

The possible numbers of strings per level is five. This is because any more strings would intersect, given a string connects two nodes. The longest possible string connects opposite nodes, and the following strings connect shorter nodes. This zig-zag pattern is copied across four levels of strings, rotated clockwise by one node each iteration. The result is the connection between 8 nodes by 20 strings that do not intersect across 4 possible intersections.

Geometric Principals

The geometry of the overall frame of the instrument is an octagon. To create this frame with wood, 22.5º cuts were made on eight pieces of wood on both sides. When bolted together, the octagonal base is made. The longest distance from one octagonal node to another was made to be 25.5”, the standard length of a guitar bridge.

The string layout of the instrument is designed to be symmetric, and have symmetrically placed pipes below string intersections. The chosen points for intersections are shown in **figure 3**.

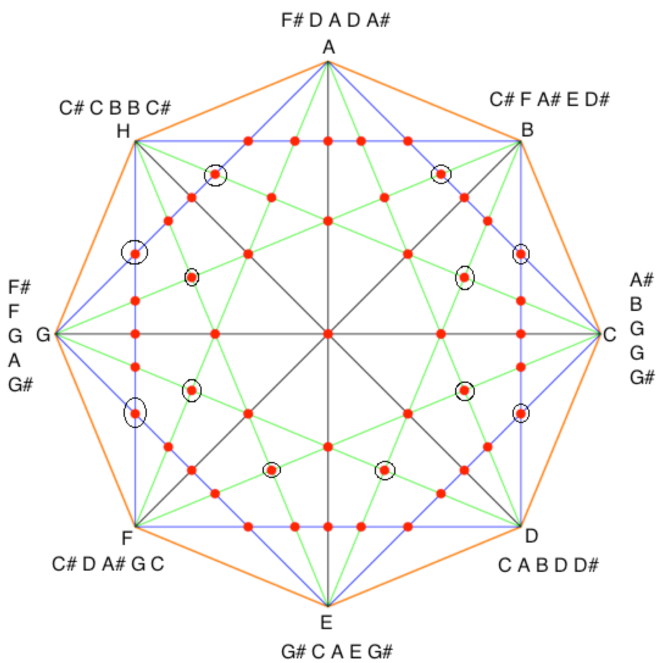

Fig 3. Intersections of strings given string notes

The notes in **figure 3** correspond to the strings connected to the corresponding nodes. The string note relates to the closest string, i.e. the string from node A to node F is the second note from the left in the list, the D note. This design allows for twelve different fifths to occur between the vertical levels of strings.

Intersection	Frequency 1 (Hz)	Frequency 2 (Hz)	Note 1	Note 2
AF-GD	587.320	439.993	d	a
AD-CF	293.660	391.989	D	g
AG-HC	369.988	493.875	F#	b
AC-BG	466.156	349.223	a#	F
AG-HF	369.988	554.356	F#	c#
AC-BD	466.156	311.122	a#	D#
BG-HE	349.223	261.621	F	C
HC-BE	493.875	329.622	b	E
GE-HF	415.298	554.356	g#	c#
CE-BD	207.649	311.122	G#	D#
HE-CF	261.621	391.989	C	g
BE-GD	329.622	439.993	E	a

Fig 4. Intersection points with frequency values

The table in **figure 4** lists the different string intersections, and the values of the notes at those intersections as well as the frequency values. The intersections are labeled with the same nodes as in **figure 3**.

The vibrating lengths of the guitar strings is required for determining the string fundamental frequency. The given 25.5" is used for the longest strings. The distance between two nodes of the octagon can be determined by the following formula, given the conversion of the diameter 22.5" is 0.5715m.

$$a = 2 \cdot r \cdot sin(\frac{\pi}{n}) = 2 \cdot 0.28575 \cdot sin(\frac{\pi}{8}) = 0.2187$$

The value a represents the side length of the octagon, the distance between two adjacent nodes. The next distance to determine is the distance between two nodes separated from a node, e.g. nodes A to C or F to H. Geometrically, this distance can be described as a triangle with two equilateral sides.

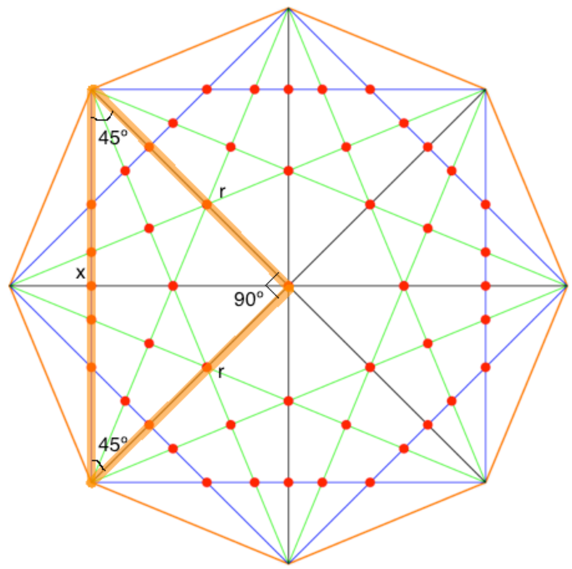

Fig 5. Example of distance x between two nodes separated by a node

This relation is seen visually in **figure 5**. The side labeled x is solved for below.

$$Hypotenuse = \frac{Opposite}{sin(\Theta)}; x = \frac{r}{sin(\Theta)} = \frac{0.28575}{sin(45)} = 0.40411$$

The final distance required is the distance between nodes separated by two nodes, e.g. A to D, and C to F. This distance is also determined by a triangle with two equilateral sides. The largest angle is composed of three exterior angles. The exterior angle in an octagon is 45º, so the largest angle in the triangle is 135º. The distance is calculated as such:

$$x = \sqrt{r^2 + r^2 - 2 \cdot r \cdot r \cdot cos(\Theta)} = \sqrt{2 \cdot 0.28575^2 - 2 \cdot 0.28575^2 \cdot cos(135)} = 0.528$$

In review, the string lengths for the guitar strings are 0.40411 m, 0.528 m, and 0.5715 m.

Acoustic Principals

Given the 20 strings used in the instrument, the frequency values for the fundamental resonance can be calculated. To determine this, the following formula is used:

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where T is the tension of the string in Newtons, μ is the linear density of the string in grams per meter, and L is the length of the vibrating string in meters. Given the desired frequency, the required tension on the string can be calculated from the other parameters. The vibrating length can be calculated from geometry, and the linear density is dependent on the guitar string gauge.

In order to determine the linear density of strings at different lengths, the linear density at 25.5” is calculated initially using the formula for fundamental frequency.

Note	Gauge (inches)	Frequency (Hz)	Tension (lbs)	Tension (N)	Density (kg/m)
e	0.010	329.600	16.2	72.061	0.0003953
b	0.013	246.900	15.4	68.503	0.0006697
g	0.017	196.000	16.6	73.840	0.0011454
d	0.026	146.800	18.4	81.847	0.0022633
A	0.036	110.000	19.5	86.740	0.0042720
E	0.046	82.900	17.5	77.844	0.0067501

Fig 6. Calculating the linear density for different gauge strings

With the linear density for each string, the tension required for the frequencies of each string can also be determined.

Frequency (Hz)	Gauge (inches)	Linear Density (kg/m)	Length (m)	Tension (N)
195.995	0.046	0.0003953	0.5715	19.838
207.649	0.046	0.0003953	0.40411	11.134
219.996	0.046	0.0003953	0.5715	24.994
233.078	0.036	0.0006697	0.5715	47.528
246.938	0.036	0.0006697	0.5715	53.349
261.621	0.036	0.0006697	0.528	51.113
277.178	0.026	0.0011454	0.40411	57.485
293.660	0.026	0.0011454	0.528	110.152
311.122	0.026	0.0011454	0.40411	72.426
329.622	0.017	0.0022633	0.528	274.224
349.223	0.017	0.0022633	0.528	307.806
369.988	0.017	0.0022633	0.40411	202.386
391.989	0.013	0.0042720	0.528	731.987
415.298	0.013	0.0042720	0.40411	481.290
439.993	0.013	0.0042720	0.528	922.246
466.156	0.010	0.0067501	0.40411	958.141
493.875	0.010	0.0067501	0.528	1835.988
523.243	0.010	0.0067501	0.40411	1207.182
554.356	0.010	0.0067501	0.40411	1355.016
587.320	0.010	0.0067501	0.528	2596.479

Fig 7. Table of tension calculations for every string

The calculated tensions for all strings in the octagonal harp are seen in **figure 7**.

The resonance frequency of a closed pipe is given as the following formula: $f_1 = \frac{c}{4L}$ where c is the speed of sound, and L is the length of the pipe. **Figure 4** displays the frequencies of the crossing strings at the points where a pipe is located. Using the greatest common denominator, it is possible to find a pipe frequency that resonates with both string frequencies.

Intersection	Frequency 1 (Hz)	Frequency 2 (Hz)	Pipe Frequency (Hz)	Pipe Length (m)
AF-GD	587.320	439.993	146.830	0.546
AD-CF	293.660	391.989	97.997	0.818
AG-HC	369.988	493.875	123.469	0.649
AC-BG	466.156	349.223	87.307	0.918
AG-HF	369.988	554.356	184.994	0.433
AC-BD	466.156	311.122	155.561	0.515
BG-HE	349.223	261.621	87.306	0.918
HC-BE	493.875	329.622	164.831	0.486
GE-HF	415.298	554.356	138.589	0.578
CE-BD	207.649	311.122	103.824	0.772
HE-CF	261.621	391.989	130.811	0.613
BE-GD	329.622	439.993	109.998	0.728

Fig 8. Pipe frequencies and lengths

The calculated pipe frequencies and lengths are given in **figure 8**. Given the pipe length parameters, enough information is given to build and construct the instrument.

Measurements

The tension of the guitar strings used was unable to be measured, as tension is inherently a difficult thing to measure. Were the instrument's strings measured, the lower frequency strings would generally be more accurate than the higher frequency strings. This conjecture holds as most guitar strings have a standard tension of around 80 N, or around 20 pounds.

The resonance frequencies of the pipes were measured by providing an impulse response to the pipe, causing the air column to excite.

Intersection	Pipe Frequency (Hz)	Pipe Length (m)	Measured Frequency (Hz)
AF-GD	146.830	0.546	144
AD-CF	97.997	0.818	95
AG-HC	123.469	0.649	121
AC-BG	87.307	0.918	83
AG-HF	184.994	0.433	182
AC-BD	155.561	0.515	154
BG-HE	87.306	0.918	85
HC-BE	164.831	0.486	161
GE-HF	138.589	0.578	136
CE-BD	103.824	0.772	101
HE-CF	130.811	0.613	128
BE-GD	109.998	0.728	107

Fig 9. Measured pipe frequencies

The measured fundamental frequencies of the pipes are given in **figure 9**. Notably the measured frequencies are all lower than the theoretical frequencies. This is most likely due to the end caps of the pipes adding a little extra length, as length is inversely proportional to the frequency of the pipe.

Improvements

A notable issue encountered while creating the instrument was the internal string tension. All strings pulled each tuning board towards the inside of the octagon, which changed the tension of all strings attached to the tuning board. To prevent this, the tuning boards had L brackets screwed into the back of them and into the base, providing a force to resist the string tension. Another issue encountered was the string sound. A lot of tinny noises occurred because the strings were vibrating next to metal L brackets. This was resolved by applying electrical tape over the brackets and all over the sound board, muffling the extra vibrations. Strings that cross the entire instrument, e.g. nodes A to E and C to G still have some extra vibration against the metal L brackets. These strings do not rest on the tuning board, which is why this problem effects only these strings. To overcome this, a small bridge is planned to be built that acts as a fulcrum.

Summary

The octagonal harp successfully illustrates the combination of a string instrument, and a instrument which uses resonance pipes. The instrument also adds in added resonance of pipes for strings that are fifths apart. The instrument is harder to play than most and has a limited range, but it sounds very interesting acoustically and has to potential to be played at high levels of skill.

Sources

[1] http://www.davidmurphyart.com/circular-harp/

[2] http://www.daddario.com/DAstringtensionguide.Page?sid=8285bee0-70f6-42b6-ae01-edd514c8165e